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MODELING OF A DIRECT-CURRENT ARC IN A PLASMATRON WITH STEP ELECTRODES

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By methods of physical modeling using a linear ridge regression it is established that in direct-current plasmatrons with step electrodes the volt-ampere characteristic of the arc is formed by heat-transfer processes, while gasdynamic factors have a slight effect. Generalized VACs of unilateral- and bilateral-emission plasmatrons in blowing the arc with air and hydrogen are refined.

Introduction. Electric-arc discharges have widespread application at present in many intensive technological processes. Electric arcs are used to produce high-temperature flows in different branches of the national economy: chemistry, metallurgy, treatment of materials, processing of waste, etc. Longitudinally blown arcs, in which high powers are attained with a rather high voltage, which makes it possible to increase the service life of plasmatrons due to the decrease in the current, are convenient for this purpose.

One of the main problems in creating plasmatrons is the calculation of their operating characteristics, for which, purpose the dependences obtained by methods of mathematical or physical modeling are suitable.

Since the use of mathematical methods of modeling for blown arcs is made difficult by the instability of discharges, one must often resort to physical modeling using experimental data. This approach also involves significant difficulties. First of all, it is necessary to isolate the dominant processes that have a fundamental effect on discharge characteristics from the set of processes. The solution of this problem was made easier recently by the possibility of using a personal computer and by the presence of rather sophisticated programs of regression analysis.

Modern programs of statistical analysis enabled us to elucidate at present the relative role of different processes in arc discharges and to refine their characteristics. We performed this work for plasmatrons with longitudinally blown arcs whose length was set by shunting processes [1]. In addition to the indicated type of plasmatrons, designs with a variable electrode diameter also have widespread application. Their advantage is in the possibility of producing regimes with ascending VACs and increasing the service life due to an increase in the surface of contact of the arc with the electrodes. The published work seeks to analyze the VACs of these plasmatrons. Apart from refinement of the characteristics, the concrete definition of the dominant mechanisms must be an important result. Comparison of the latter in different types of plasmatrons will make it possible to reveal the effect of structure features on the character of the processes in the arc.

1. Investigation Procedure. Similar investigations for a longitudinally blown arc in plasmatrons with smooth electrodes were performed earlier [1-5]. In the present work, we used experimental data of our own investigations on bilateral-emission plasmatrons with a variable electrode diameter. Air and hydrogen were used as working gases. In operation with air, its flow rate changed from $G = 14$ g/sec to $G = 356$ g/sec while the geometric characteristics of a discharge chamber varied within the following limits: the diameter and length of the diaphragm (both diaphragms – the anode diaphragm and the cathode diaphragm – were equal in length and in diameter) $d = 5$; 8; 10; 14 mm, $l = 25$; 30; 50; 69; 100 mm; the diameter of the electrodes (also the same for the anode and the cathode) $D = 26$; 27; 30; 35; 40 mm; the length of the electrodes $L = 180$ mm. In operation with hydrogen, the corresponding parameters varied within the limits: $d = 8$; 16 mm; $l = 12$; 25; 30; 64; 96 mm; $D = 27$; 35 mm; $G = 2$; 4 g/sec.

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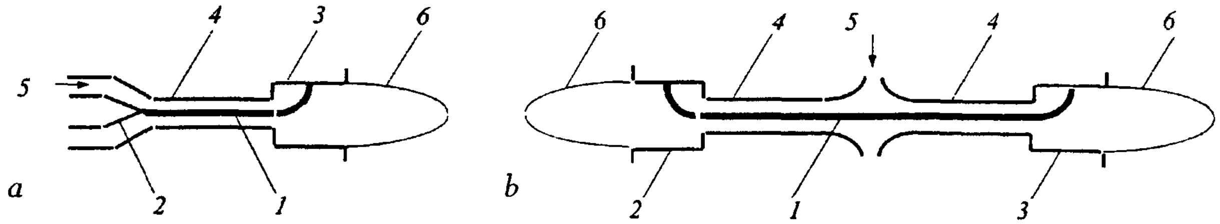

Fig. 1. Diagrams of direct-current arc plasmatrons with step electrodes in unilateral emission (a) and bilateral (b) emission: 1) electric arc; 2) cathode; 3) anode; 4) diaphragm; 5) injection of heated gas; 6) emission of plasma jet.

For analysis of the characteristics of a unilateral-emission plasmatron with a variable electrode diameter, we used the experimental data of [6]. The plasmatron had a zirconium cathode and a copper step anode with d = 22.5; 25 mm; Z = 140; 180; 220; 290 mm; D = 41.6; 46 mm; L = 210; 280; 320; 360 mm; air at G = 30; 50; 70; 90 g/sec was used for operation.

Diagrams of the investigated plasmatrons are shown in Fig. 1. In the unilateral-emission plasmatron (Fig. 1a), the arc 1 is initiated between the hot rod cathode 2 and the cooled copper anode 3. Its main portion is located in the narrowed part of the anode (diaphragm) 4. The heated gas is supplied with a swirl to the diaphragm on the source side of the cathode (arrow 5 in Fig. 1a) while the hot gas is emitted from the anode in the form of a plasma jet 6. In the bilateral-emission plasmatrons, (Fig. 1b), both electrodes are identical. They are copper tubes and have expanded parts (the electrodes – the cathode 2 and the anode 3) and diaphragms 4 of a smaller diameter. The heated gas 5 is supplied between the diaphragms while the plasma jets 6 are emitted in opposite directions. Experimental data were processed using programs of multiple linear regression from "Statistics for Windows." The dominant mechanisms were selected by a step-by-step "forward" method for the threshold value of the Fisher variance ratio: for direct and inverse analyses $F_{\text{dir}} = 4.00$ and $F_{\text{inv}} = 3.99$. Examination showed a strong mutual correlation between the dominant processes of heat transfer. Therefore, to reveal their relative role, we used a ridge regression, the degree of influence of the individual processes being determined from the standardized coefficients of regression β.

In this work, we analyzed the volt-ampere characteristics, which are fundamental for an arc discharge, since they govern both the load (in the electric circuit) and energy-transfer parameters of the arc. These characteristics were approximated by power-law dependences in nongeneralized

$$U = CI^{\alpha} d^{\beta} l^{\gamma} D^{\delta} G^{\varepsilon} \tag{1}$$

and generalized

$$\pi_{\text{dep}} = C \prod_{i} \pi_{\text{ind}}^{B_i} \tag{2}$$

forms.

These expressions were represented in natural logarithms for reduction to the form of a multiple linear regression, after which the constant factors $c = \ln C$ and the natural coefficients α, β, γ, δ, ε, and B_i, which are the exponents in (1) and (2), were calculated.

We used as dependent generalized variables the generalized resistance of the are determined from the diameter of the diaphragm $\pi_{\text{dep}d} = Ud\sigma_0/I$ and from its length $\pi_{\text{dep}l} = Ul\sigma_0/I$ as well as from the electrode diameter $\pi_{\text{dep}D} = UD\sigma_0/I$. The initial set of independent generalized variables involved: $\pi_{\text{conv}d} = \sigma_0 h_0 Gd/I^2$, $\pi_{\text{conv}l} = \sigma_0 h_0 GD/I^2$, and $\pi_{\text{conv}D} = \sigma_0 h_0 GD/I^2$, i.e., the similarity numbers for convective heat transfer determined from diameter, the length of the diaphragm, or the electrode diameter, respectively; $\pi_{\text{cond}d} = \sigma_0 \lambda_0 T_0 d^2/I^2$, $\pi_{\text{cond}l} = \sigma_0 \lambda_0 T_0 l^2/I^2$, and $\pi_{\text{cond}D} = \sigma_0 \lambda_0 T_0 D^2/I^2$, i.e., the similarity numbers for conductive heat transfer, $\pi_{\text{rad}d} = \sigma_0 Q_0 d^4/I^2$, $\pi_{\text{rad}l} = \sigma_0 Q_0 l^4/I^2$, and $\pi_{\text{rad}D} = \sigma_0 Q_0 D^4/I^2$, i.e., the similarity numbers for heat transfer by radiation in the volume approximation; $\pi_{\text{turb}d} = \sigma_0 \rho_0 h^{1.5} d^3/I^2$, $\pi_{\text{turb}l} = \sigma_0 \rho_0 h^{1.5} l^3/I^2$, and $\pi_{\text{turb}D} = \sigma_0 \rho_0 h^{1.5} D^3/I^2$, i.e., the

TABLE 1. Standardized Coefficients of Multiple Linear Ridge Regression for VACs of Plasmatrons with Step Electrodes

Type of plasmatron and working gas	Standardized coefficients β for variables								
	$\pi_{\mathrm{conv}l}$	$\pi_{\mathrm{conv}d}$	$\pi_{\mathrm{conv}D}$	$\pi_{\mathrm{cond}l}$	$\pi_{\mathrm{cond}d}$	$\pi_{\mathrm{turb}l}$	$\pi_{\mathrm{rad}l}$	$\mathrm{Re}_{\mathrm{hot}d}$	$\pi_{\mathrm{turb}d}$
Bilateral emission; air; $N=282$	0.257	0.164	0.196	0.207	0.081	0.141	0.102	0.119	–
Unilateral emission; air; $N=84$	0.202	–	0.159	0.203	–	0.201	0.195	0.122	–
Bilateral emission; hydrogen; $N=29$	–	–	–	0.386	0.382	–	–	–	0.349

similarity numbers for heat transfer with thermal turbulence; $\pi_{\mathrm{conv}I} = b\mu_0\rho_0\sigma_0^2h_0^2D^4/I^2$, i.e., the similarity number for convective heat transfer with the motion of the arc in the intrinsic magnetic field (rotation of the terminal segments of the arc in the electrodes, on which solenoids series-connected to the arc are installed); $\mathrm{Re}_{\mathrm{cold}d} = G/\eta_{0\,\mathrm{cold}}d$; $\mathrm{Re}_{\mathrm{cold}l} = G/\eta_{0\,\mathrm{cold}}l$; $\mathrm{Re}_{\mathrm{hot}d} = G/\eta_{0\,\mathrm{hot}}d$; $\mathrm{Re}_{\mathrm{hot}l} = G/\eta_{0\,\mathrm{hot}}l$, i.e., the Reynolds numbers for the cold (blowing the arc) and hot (blowing through the arc) flows determined from the diameter and length of the diaphragm, respectively.

In analysis of VACs in a nongeneralized form, the arc voltage was used as the dependent variable while the parameters indicated in the right-hand side of Eq. (1) – I, d, l, D, and G – were the independent variables. The exponents in the nongeneralized expression can also be determined from the exponents of the generalized formula and compared. This comparison makes it possible to evaluate the reliability of selecting the independent generalized similarity numbers.

2. Analysis of the Data. The standardized coefficients on the independent generalized parameters selected according to the program of ridge regression for a bilateral-emission plasmatron when air and hydrogen are used as working gases and the data for an air-blown arc in a unilateral-emission plasmatron are given in Table 1.

As can be seen from Table 1 the gasdynamic factors have a slight effect on the VACs of arc discharge. In the air plasma, the relative contribution of the Reynolds number is on the order of 10%. The processes of energy transfer are of first importance. Convective heat transfer is leading among them in the air plasma. Its fraction in the bilateral-emission plasmatron is equal to approximately 50%, and, in the case of a unilateral-emission plasmatron, it attains 33%. It turns out that convective heat transfer is affected more strongly by the length of the diaphragm than by its diameter.

The role of conduction in the air plasma is found to rank second. In the case of bilateral emission, this mode of heat transfer is more pronounced (23%) than in the case of unilateral emission (19%). It is affected mainly by the diaphragm length while the effect of the diameter is slight. Turbulence that manifests itself somewhat more strongly than radiant energy transfer ranks third among the processes of heat transfer. In practice, the turbulence is affected only by the diaphragm length, its action being more substantial (19%) in the unilateral-emission plasmatron than in plasma emission in two directions (11%).

In just the same way, the role of radiation in unilateral emission is twice as efficient as in bilateral emission. The intensification of radiant heat transfer in a unilateral-emission plasmatron can be attributable to the presence in the plasma of the material of the rod cathode, which is absent in the case of bilateral emission. The significant effect of the turbulence in the plasmatron with a rod-shaped cathode is possibly determined by an increase in the disturbance of the gas flow by this electrode. It should be noted that the role of turbulent energy transfer in plasmatrons with step electrodes is less significant than in the case of smooth electrodes, where periodic shunting of the arc disturbs strongly the flow of the heated gas.

The processes in the electrodes have a small effect as compared to the diaphragm, in which the gas goes through a narrow channel. Of the various mechanisms dependent on the electrode diameter, only convection affects noticeably the VACs in the air plasma. Its role is approximately the same in both variants of plasma emission (15%). The criterion of convective heat transfer due to the rotation of the terminal segment of the arc in the

TABLE 2. Standardized Coefficients of Ordinary Multiple Linear Regression for VACs of Plasmatrons with Step Electrodes

Type of plasmatron and working gas	Standardized coefficients β				
	$\pi_{\mathrm{conv}l}$	$\pi_{\mathrm{conv}D}$	$\pi_{\mathrm{cond}l}$	$\pi_{\mathrm{cond}d}$	$\pi_{\mathrm{rad}l}$
Bilateral emission; air; $N = 282$	0.634	0.180	0.271	–	–
Unilateral emission; air; $N = 84$	0.656	–	–	–	0.358
Bilateral emission; hydrogen; $N = 29$	–	–	–	0.811	0.389

TABLE 3. Correlation Matrix for Generalized VACs of Bilateral-Emission Plasmatron, Air

Variables	Coefficients of correlation between variables								
	$\pi_{\mathrm{conv}d}$	$\pi_{\mathrm{cond}d}$	$\pi_{\mathrm{hot}d}$	$\pi_{\mathrm{conv}D}$	$\pi_{\mathrm{conv}l}$	$\pi_{\mathrm{rad}l}$	$\pi_{\mathrm{turb}l}$	$\pi_{\mathrm{cond}l}$	π_{dep}
$\pi_{\mathrm{conv}d}$	1.00	0.66	0.54	0.96	0.86	0.11	0.17	0.27	0.80
$\pi_{\mathrm{cond}d}$	0.66	1.00	−0.28	0.57	0.60	0.39	0.46	0.59	0.66
$\mathrm{Re}_{\mathrm{hot}d}$	0.54	−0.28	1.00	0.59	0.43	−0.29	−0.30	−0.32	0.28
$\pi_{\mathrm{conv}D}$	0.96	0.57	0.59	1.00	0.81	0.00	0.07	0.20	0.75
$\pi_{\mathrm{conv}l}$	0.86	0.60	0.43	0.81	1.00	0.56	0.60	0.66	0.96
$\pi_{\mathrm{rad}l}$	0.11	0.39	−0.29	0.00	0.56	1.00	0.99	0.95	0.61
$\pi_{\mathrm{turb}l}$	0.17	0.46	−0.30	0.07	0.60	0.99	1.00	0.98	0.66
$\pi_{\mathrm{cond}l}$	0.27	0.59	−0.32	0.20	0.66	0.95	0.98	1.00	0.72
π_{dep}	0.80	0.66	0.28	0.75	0.96	0.61	0.66	0.72	1.00

electrode in the interaction of the current with the magnetic field produced by the series-connected solenoid is also not included in the selected criteria.

Comparison of the characteristics of the arc blown with air and hydrogen shows a significant role of the properties of the working gas. It turned out that conductive heat transfer, which depends to the same extent on the diameter and length of the diaphragm, is of major importance. The effect of turbulence is also more substantial as compared to air. The relative effect of turbulence is approximately one third of the total effect; unlike the air plasma, the action of turbulence depends on the diameter of the diaphragm rather than on the length. No processes are noted in the hydrogen arc other than conductive and turbulent heat transfer. The processes associated with entry of the electrode material into the plasma have no effect on the VACs either.

It is of interest to compare the generalizations by the methods of ordinary regression and ridge linear regression. Table 2 shows the results of selecting the dominant processes by means of ordinary regression that does not use a shift of the coefficients.

From Table 2 it can be seen that ordinary regression selects a much smaller number of independent variables than ridge regression: only three criteria turned out to be effective in the case of a bilateral-emission plasmatron in blowing the arc with air, and two variables were used in each of the other two cases. It is precisely these processes that were among the dominant processes and were the most efficient in the ridge regression. The VAC of the air arc depends by two thirds on convective heat transfer while conduction is dominant in the hydrogen arc as before. Turbulent heat transfer disappeared from the selected criteria, but the role of radiation became enhanced, especially in the hydrogen arc, for the VAC of which its role is not established at all in the case of ridge regression.

This character of selecting the independent variables in ordinary regression is caused by the strong correlation between them. In this case, even the efficient processes are not among the selected processes if they are strongly correlated with a certain more significant mechanism. The influence of the related competing effects is suppressed and their independent role in no way reveals itself. Accordingly, relatively weak factors are more pronounced. This is easily seen from the data of correlation matrices.

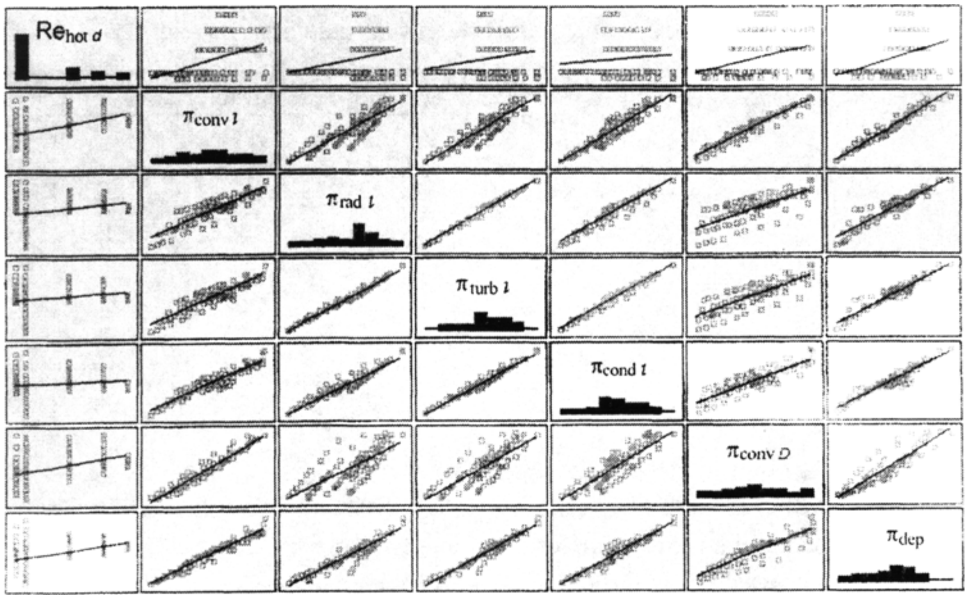

Fig. 2. Matrix of diagrams of scatter of experimental points for different pairs of variables for unilateral-emission plasmatron.

Table 3 gives this matrix for the bilateral-emission plasmatron in blowing the arc with air while Fig. 2 shows the matrix of the diagrams of scatter of experimental points for different pairs of variables that provide clearly information on the mutual correlation.

The table shows that the most efficient influence on VAC is exerted by the similarity numbers, which are more closely related to the dependent variables:

$\pi_{\text{conv}l}$: $k = 0.96$; $\beta_{\text{R}} = 0.257$; $\beta_{\text{ord}} = 0.634$;
$\pi_{\text{conv}l}$: $k = 0.72$; $\beta_{\text{R}} = 0.207$; $\beta_{\text{ord}} = 0.271$;
$\pi_{\text{conv}D}$: $k = 0.75$; $\beta_{\text{R}} = 0.196$; $\beta_{\text{ord}} = 0.180$.

The remaining similarity numbers except $\pi_{\text{conv}d}$ have smaller k and β_{R} and are correlated better with more efficient numbers:

$\pi_{\text{turb}l} \to \pi_{\text{cond}l}$, $k = 0.98$:
$\text{Re}_{\text{hot}} \to \pi_{\text{conv}D}$, $k = 0.59$;
$\pi_{\text{rad}l} \to \pi_{\text{cond}l}$, $k = 0.95$;
$\pi_{\text{cond}d} \to \pi_{\text{cond}l}$, $k = 0.59$.

The variable $\pi_{\text{conv}d}$ has a strong correlation with the dependent variable ($k = 0.8$) but it is correlated even more significantly with the number $\pi_{\text{conv}l}$ ($k = 0.86$), which, in turn, is also related significantly to the dependent variable.

A similar situation is obtained in the case of unilateral emission, the mutual correlations for which are presented in Fig. 2. It can be seen that the smallest scatter for the correlation with the independent variable is yielded by $\pi_{\text{conv}l}$ and $\pi_{\text{rad}l}$. At the same time, $\pi_{\text{conv}D}$, $\pi_{\text{cond}l}$, and $\pi_{\text{turb}l}$ are closest to $\pi_{\text{conv}l}$ while for $\pi_{\text{turb}l}$ and $\pi_{\text{cond}l}$ we observe a pronounced correlation with $\pi_{\text{rad}l}$ that is also selected in the variables for regression. The Reynolds number is related weakly to all the variables including $\pi_{\text{dep}d}$ but is correlated well with $\pi_{\text{conv}l}$.

In the case of the hydrogen arc, we omitted only the one variable $\pi_{\text{turb}d}$ that made the smallest contribution to VAC formation.

Generalization of experimental data in the form of empirical formulas is also of certain interest. In this case, analysis enables us to clearly elucidate the relative role and character of the effect of the individual variables (Table 4).

The data of Table 4 for empirical formulas show that in blowing the arc with air the step electrodes make it possible to obtain ascending VACs, which ensures more stable operation of the setup and improvement of its efficiency. It can also be seen that variations in the flow rate of the gas and the length of the diaphragm have the

TABLE 4. Slope of the Dependence of Arc Voltage on Different Parameters

Type of plasmatron and working gas	Method of representation of VAC	Slope of characteristic for different parameters				
		I	G	d	l	D
Bilateral emission; air	Generalized	0.142	0.335	−0.913	0.420	0.087
	Empirical	0.107	0.341	−0.957	0.418	−0.243
Unilateral emission; air	Generalized	0.302	0.269	–	0.640	–
	Empirical	0.291	0.321	–	0.773	–
Bilateral emission; hydrogen	Generalized	−0.014	0.000	−0.107	0.274	0.000
	Empirical	0.000	0.000	0.000	0.283	0.000

most substantial positive effect on the arc voltage. The slope of the increase in the VAC as a function of the current and the diaphragm length turned out to be higher in the plasmatron with a rod electrode, but the degree of influence of the gas flow rate decreased somewhat in this case as compared to bilateral emission. In the bilateral variant a strong decrease in voltage is observed as the diaphragm diameter increases, while for the plasmatron with a rod cathode we were unable to establish an effect of the diaphragm diameter, since it remained practically constant. In the case of the hydrogen arc, variation of the geometric parameters of the discharge chamber was quite perceptible but only the diaphragm length had a strong effect on the VAC of the discharge. A variation in the size of the opening of the diaphragms and the electrodes had no effect on the characteristic, as did a variation in the current. Therefore, the VAC of the hydrogen arc was gently sloping, in practice, and reaching its ascending branch turned out to be difficult.

We can also determine the relative effect of different primary variables from the generalized characteristics, summing the exponents on each of them. Table 4 compares the slope of the dependence of the voltage on different parameters for generalized and empirical expressions.

Table 4 shows that there is a difference between a generalized VAC and an empirical VAC concerning the functional relation of the arc voltage to the current and the diameter of the electrode in the bilateral-emission plasmatron in operation with air. This means that, in this case, a certain significant number affected by the current and the diameter of the electrode but not by the flow rate of the gas is not included in the initial set of generalized independent variables. The difference between the generalized and empirical characteristics in the effect of the diaphragm dimensions on the arc voltage turned out to be insignificant. This shows that the selected similarity numbers describe rather completely the processes in the diaphragm.

Since in the experiments with the plasmatron with a rod cathode the diameter of the diaphragm and the electrode changed little, their effect was not reflected in the VACs. Comparison of the voltages as functions of the current and the dimensions of the diaphragm for the generalized and empirical VACs shows that there is a small disagreement in this case. With weaker stabilization of the arc in the diaphragm of the unilateral-emission plasmatron as compared to the bilateral-emission plasmatron the effect of an eddy of cold gas flow apparently manifests itself in the former. This phenomenon, however, is not represented by a criterion in the initial set, since the experimental data do not carry the information required.

3. Volt-Ampere Characteristics. Volt-ampere characteristics of the discharge are formed from data for natural coefficients of multiple regression. Table 5 gives the natural coefficients in generalized formulas for two methods of processing the results of the experiment.

The natural coefficients (Table 5) correspond to the exponents on the indicated variables of the generalized VAC (formula 2) while the constant coefficient of regression is a natural logarithm of the coefficient of VAC: $c = \ln C$.

The exponents for the variables of the empirical formula are given in Table 4, the constant factor of VAC having the values:

a bilateral-emission plasmatron, air $C = 62.74$;
a unilateral-emission plasmatron, air $C = 53.73$;

TABLE 5. Natural Coefficients of Regression for Different Variables

Type of plasmatron and working gas	Type of regression	Variables									
		c	$\pi_{\text{conv}d}$	$\pi_{\text{conv}l}$	$\pi_{\text{conv}D}$	$\pi_{\text{cond}d}$	$\pi_{\text{cond}l}$	$\pi_{\text{turb}d}$	$\pi_{\text{turb}l}$	$\pi_{\text{rad}l}$	Re_d
Bilateral emission; air	Ridge	0.725	0.072	0.109	0.087	0.041	0.068	–	0.033	0.019	0.067
	Ordinary	1.276	–	0.268	0.080	–	0.089	–	–	–	–
Unilateral emission; air	Ridge	0.371	–	0.081	0.073	–	0.078	–	0.065	0.052	0.115
	Oridinary	0.409	–	0.264	–	–	–	–	–	0.096	–
Bilateral emission; hydrogen	Ridge	3.246	–	–	–	0.217	0.137	0.153	–	–	–
	Ordinary	3.582	–	–	–	0.460	–	–	–	0.073	–

a bilateral-emission plasmatron, hydrogen $C = 356.74$.

The ridge regression is convenient because it establishes the relative roles of different processes that occur in the arc discharge, which is of great cognitive significance. However, the number of independent variables can prove to be too great, and the formula acquires a cumbersome form. For example, for a bilateral-emission plasmatron that operates with air the VAC obtained by the ridge regression is represented by the expression

$$\frac{Ud\sigma_0}{I} = 2.065 \left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{0.072} \left(\frac{Gl\sigma_0 h_0}{I^2}\right)^{0.109} \left(\frac{GD\sigma_0 h_0}{I^2}\right)^{0.087} \left(\frac{\sigma_0 \lambda_0 T_0 d^2}{I^2}\right)^{0.041} \times$$

$$\times \left(\frac{\sigma_0 \lambda_0 T_0 l^2}{I^2}\right)^{0.068} \left(\frac{\rho_0 \sigma_0 h_0^{1.5} l^3}{I^2}\right)^{0.033} \left(\frac{Q_0 \sigma_0 l^4}{I^2}\right)^{0.019} \left(\frac{G}{\eta_{\text{hot}} d}\right)^{0.067} \tag{3}$$

The standard error of the natural logarithm of this expression is SE = 0.123. Therefore, formulas obtained by the ordinary regression can prove to be more convenient for practical purposes. In addition to a reduction in the number of independent variables, they have a higher accuracy, since the calculation is performed by the least-squares method without a shift of the coefficients.

Given below are the corresponding formulas obtained from the data of Table 5 (for the ordinary regression): a bilateral-emission plasmatron that operates with air:

$$\frac{Ud\sigma_0}{I} = 3.582 \left(\frac{Gl\sigma_0 h_0}{I^2}\right)^{0.268} \left(\frac{GD\sigma_0 h_0}{I^2}\right)^{0.080} \left(\frac{\sigma_0 \lambda_0 T_0 l^2}{I^2}\right)^{0.089}, \quad \text{SE} = 0.103\,; \tag{4}$$

a plasmatron with a rod cathode that operates with air:

$$\frac{Ud\sigma_0}{I} = 1.505 \left(\frac{Gl\sigma_0 h_0}{I^2}\right)^{0.264} \left(\frac{Q_0 \sigma_0 l^4}{I^2}\right)^{0.096}, \quad \text{SE} = 0.059\,; \tag{5}$$

a bilateral-emission plasmatron that operates with hydrogen

$$\frac{Ud\sigma_0}{I} = 35.95 \left(\frac{\sigma_0 \lambda_0 T_0 d^2}{I^2}\right)^{0.460} \left(\frac{Q_0 \sigma_0 l^4}{I^2}\right)^{0.073}, \quad \text{SE} = 0.078\,. \tag{6}$$

The characteristic values of plasma properties used in these qeneralized expressions are obtained by the method of their temperature dependence [7]. As compared to them, the empirical formulas turn out to be more cumbersome but somewhat more exact: the bilateral-emission plasmatron operating with air

$$U = 62.74 I^{0.107} G^{0.341} d^{-0.957} D^{-0.243} l^{0.418}, \quad \text{SE} = 0.097\,;$$

the unilateral-emission plasmatron operating on air

$$U = 53.73 I^{0.291} G^{0.321} l^{0.773} , \quad \mathrm{SE} = 0.051 .$$

Conclusions. The analysis of the experimental data on the VACs of a direct-current arc discharge in plasmatrons with step electrodes by the methods of linear ridge regression showed that the leading part in the formation of the characteristics of the arc is played by heat-transfer processes. We also note a role of the Reynolds number, but it is not as significant as is generally assumed (the relative contribution is $\sim 10\%$). It is also shown that the processes of heat transfer are correlated strongly. This enables us to simplify the generalized VACs by using only the most effective criteria. When the arc is blown with air, connective and conductive heat transfers are of great significance while turbulence manifests itself more weaking in stabilized discharges. In a hydrogen arc, turbulent heat transfer has a substantial effect, conductive heat removal being the most pronounced.

It has been established that in strong contraction of the discharge by the diaphragm walls its length and the flow rate of the gas have an effect on the VACs while the dependence on the diameter of the diaphragm is weaker. The processes in the extended part of the electrodes also prove to be insignificant.

NOTATION

B_i, natural coefficient of regression; c, constant coefficient of regression; $c = \ln C$; C, constant factor of VAC; D, electrode diameter; d, diaphragm diameter; N, number of experimental points; F, Fisher variance ratio; G, flow rate of heated gas; h, enthalpy; I, current; L, electrode length; l, diaphragm length; Q, plasma emissivity; Re, Reynolds number; SE, standard error of regression; T, temperature; U, arc voltage; k, coefficient of correlation; β, standardized coefficient of regression; η, dynamic-viscosity factor; λ, thermal conductivity; π, similarity number; ρ, density; σ, electrical conductivity. Subscripts: R, ridge regression; hot, hot; dep, dependence; conv, convective; cond, conductive; rad, radiant; ord, ordinary regression; inv, inverse; dir, direct; turb, turbulent; cold, cold; 0, characteristic value of property; ind, independent; i, number of natural coefficient of regression; α, β, γ, δ, and ε, exponents.

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